Name:

Spring 2010 Math 245-2 Exam 3

Please read the following directions:
Please write legibly, with plenty of white space. Please put your answers in the designated areas. To get credit, you must also show adequate work to justify your answers. If unsure, show the work. All problems are worth 5-10 points. You may use your book and/or notes, but no calculators or other aids. This exam will last 60 minutes; pace yourself accordingly. If you are done early, you may leave – but NOT during the last five minutes of the exam, during which you are asked to remain quiet and in your seat. Good luck!

Problem	Min Score	Your Score	Max Score
1.	5		10
2.	5		10
3.	5		10
4.	5		10
5.	5		10
6.	5		10
7.	5		10
8.	5		10
9.	5		10
10.	5		10
Total:	50		100

Problem 1. Carefully define each of the following terms:

a. cardinal number

b. ordinal number

c. binary relation

d. union

e. injective

Problem 2. For all sets A, B, C, prove that $A \cap B \cap C \subseteq A \cup C$.

Problem 3. Let $A = \{a, b, c\}, B = \{b, c, d\}$. Find $\mathcal{P}(A) \cap \mathcal{P}(B)$.

Problem 4. For $A = \{a, b\}$, find a relation R on A that is not reflexive and not symmetric.

Problem 5. Fix $A = \{1, 2, 3\}$. We define relation R on A via xRy if and only if $x + 3 < y^2$. Determine, with proof, whether R is a partial order.

Problem 6. Fix $A = \{1, 2, 3\}$. We define relation R on A via xRy if and only if $x + 3 < y^2$. Determine, with proof, whether R is a function.

Problem 7. Let $f : X \to Y, g : Y \to Z$ be functions, with $g \circ f$ injective. Prove or disprove that f is injective.

Problem 8. Prove that any arrangement of five points in a unit square will have some pair of them within 0.75 of each other.

Problem 9. Find a finite-state automaton on alphabet $\{a, b\}$ that recognizes all strings with an even number of b's.

Problem 10. Solve the recurrence $a_n = a_{n-1} + 6a_{n-2}$, with initial conditions $a_0 = 4, a_1 = 7$.